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[This question paper contains 7 printed pages]

Your Roll No. : ....

Sl. No. of Q. Paper : **1823** **GC-4**

Unique Paper Code : 32351201

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Real Analysis

Semester : II

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(c) **All** questions are compulsory.

(b) Attempt any **two** parts from each question.

1. (a) Define Infimum and Supremum of a non-empty subset of R.

Find infimum and supremum of the set

$$S = \left\{1 - \frac{(-1)^n}{n} : n \in N\right\}.$$ 5

P.T.O.

(b) Prove that a number u is the supremum of a non-empty subset S or R if and only if :

(i) $S \leq u \quad \forall s \in S$.

(ii) For any $\in > 0$, there exists $s_\in \in S$ such that $u - \in < S_\in$. 5

(c) State Archimedean Property of Real numbers. Prove that if $S = \left\{\frac{1}{n} : n \in N\right\}$, then inf S=0. 5

2. (a) Let A and B be bounded non-empty subsets of R. Define :

$A + B = \{a + b : a \in A \text{ and } b \in B\}$

Prove that inf (A+B) = inf A+inf B. 5

(b) State Density Theorem. Show that if x and y are real numbers with x < y, then there exists an irrational number z such that $x < z < y$. 5

(c) Define limit point of a set. Find limit points of $]0, 1[$. 5

3. (a) Define the convergence of a sequence (x_n) of real numbers. Show that if (x_n) is a convergent sequence of real numbers such that $x_n \geq 0 \; \forall n \in N$, then $x = \lim x_n \geq 0$.

(b) Using the definition of the limit of a sequence, find the following limits :

(i) $\lim_{n\to\infty}\left(\frac{3n+1}{2n+5}\right)$.

(ii) $\lim_{n\to\infty}\left(\frac{\sqrt{n}}{n+1}\right)$. 5

(c) Prove that $\lim_{n\to\infty} n^{1/n} = 1$.

5

4. (a) Let (x_n) be a sequence of real numbers that converges to x and suppose that $x_n \geq 0 \forall n \in N$. Show that the sequence $\sqrt{x_n}$ converges to $\sqrt{x}$. 5

(b) Prove that every monotonically increasing bounded above sequence is convergent. 5

(c) If $x_1 < x_2$ are arbitrary real numbers and $x_n = \frac{1}{2}(x_{n-2} + x_{n-1})$ for n>2, show that (x_n) is convergent. What is its limit ? 5

5. (a) Define a Cauchy Sequence. Is the sequence (x_n) a Cauchy Sequence, where $x_n = 1 + \frac{1}{2!} + \ldots\ldots + \frac{1}{n!}$? Justify your answer. $7\frac{1}{2}$

(b) State and prove Bolzano Weierstrass Theorem for sequences. Justify the theorem with an example. $7\frac{1}{2}$

(c) (i) Show that if (x_n) is unbounded, then there exists a subsequence (x_{nk}) such that : $\lim\left(\frac{1}{x_{nk}}\right)=0$. 5

(ii) Show that the sequence $\left(1, \frac{1}{2}, 3, \frac{1}{4}, \ldots\ldots\right)$ is divergent.

$2\frac{1}{2}$

6. (a) If $\sum_{n=1}^{\infty} x_n$ converges, then prove that $\lim_{n\to\infty} x_n = 0$. Does the converse hold ?

5

(b) Test the convergence of any **two** of the following series : 5

(i) $\Sigma \frac{n+1}{n2^n}$

(ii) $\Sigma\left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$

P.T.O.

(iii) $\sum \frac{n^2}{n!}$

(c) State the Alternating Series Test. Show that the alternating series $\sum \frac{(-1)^n}{n}$ is convergent. 5

7. (a) Let $0 \leq a_n \leq b_n \forall n$. Show that : 5

(i) If $\sum_{n=1}^{\infty} b_n$ converges, then so does $\sum_{n=1}^{\infty} a_n$.

(ii) If $\sum_{n=1}^{\infty} a_n$ diverges, then so does $\sum_{n=1}^{\infty} b_n$.

(b) Show that every absolutely convergent series is convergent but the converse is not true. 5

(c) State Integral Test. Find the condition of convergence of the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n^p}$. 5

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Your Roll No. : ....

Sl. No. of Q. Paper : **1824** GC-4

Unique Paper Code : 32351202

Name of the Course : **B.Sc.(Hons.) Mathematics–I**

Name of the Paper : Differential Equations

Semester : II

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Use of non-programmable scientific calculator is allowed.

SECTION – A

1. Attempt any **three** parts, each part is of **5** marks.

(a) Solve the initial value problem :

$$x\frac{dy}{dx} + y = xy^{3/2},\ y(1)=4.$$

(b) Determine the most general function M (x, y) such that the equation

M (x, y) dx + $(x^2 y^3 + x^4 y)$ dy =0, is exact and hence solve it.

P.T.O.

(c) Solve the differential equation :

$(x^2 - 3y^2)\, dx + 2xy dy = 0.$

(d) Check the exactness of the differential equation :

$(3y + 4xy^2)\, dx + (2x + 3x^2 y)\, dy = 0.$

Hence solve it by finding the integrating factor of the form $x^p y^q$.

2. Attempt any **two** parts; each part is of **5** marks.

(a) A certain moon rock was found to contain equal numbers of potassium and argon atoms. Assume that all the argon is the result of radioactive decay of potassium (its half-life is about 1.28×10^9 years) that one of every nine potassium atom disintegrations yields an argon atom. What is the age of the rock, measured from the time it contained only potassium ?

(b) A hemispherical bowl has top radius 4 ft and at time t = 0 is full of water. At that moment a circular hole with diameter 1 inch is opened in the bottom of the tank. How long will it take for all the water to drain from the tank ?

(c) A motor boat starts from rest (initial velocity v (0) = v_0 =0). Its motor provides a constant acceleration of 4 ft/s², but water resistance causes a deceleration of $\frac{v^2}{400}$ ft/s². Find v when t =10 s, and also find the limiting velocity as t → + ∞ (that is, the maximum possible speed of the boat).

SECTION – B

3. Attempt any **two** parts; each part is of **7.5** marks.

(a) Consider the American system of two lakes: Lake Erie feeding into Lake Ontario. Assuming that volume in each lake to remain constant and that Lake Erie is the only source of pollution for Lake Ontario.

(i) Write down a differential equation describing the concentration of pollution in each of two lakes, using the variables V for volume, F for flow, c(t) for concentration at time t and subscripts 1 for Lake Erie and 2 for Lake Ontario.

(ii) Suppose that only unpolluted water flows into Lake Erie. How does this change the model proposed ?

(iii) Solve the system of equations to get expression for the pollution concentration $c_1(t)$ and $c_2(t)$.

(b) The following model describes the levels of a drug in a patient taking a course of cold pills :

$$\frac{dx}{dt} = I - k_1 x, \quad x(0) = 0$$

$$\frac{dy}{dt} = k_1 x - k_2 y, \quad y(0) = 0$$

Where k_1 and k_2 ($k_1 > 0$, $k_1 > 0$ and $k_1 \neq k_2$) describes rate at which the drug moves between the two sequential compartments (the GI-tract and the bloodstream) and I denotes the amount of drug released into the GI-tract in each step. At time t, x and y are the levels of the drug in the GI-tract and bloodstream respectively.

(i) Find solution expressions for x and y which satisfies this pair of differential equations.

(ii) Find the levels of the drug in the GI-tract and the bloodstream as $t \to \infty$.

(c) In view of the potentially disastrous effects of overfishing causing a population to become extinct, some governments impose quotas which vary depending on estimates of the population at the current time. One harvesting model that takes this into account is

$$\frac{dX}{dt} = rX\left(1 - \frac{X}{K}\right) - h_0 X.$$

(i) Find the non-zero equilibrium population.

(ii) At what critical harvesting rate can extinction occur ?

SECTION – C

4. Attempt any **four** parts; each part is of **5** marks.

(a) Use the method of variation of parameters to find a particular solution of the differential equation

$$y'' - 4y' + 4y = 2e^x.$$

(b) Use the method of undetermined coefficients to solve the differential equation

$$y''+y=\sin x.$$

(c) A body with mass $m=\frac{1}{2}$ kg is attached to the end of the spring that is stretched 2 m (meters) by a force of 100 N (Newtons). It is set in motion with initial position $x_0 = 1$ m and initial velocity $v_0 = -5$m/s. Find the position function of the body as well as the amplitude, frequency and period of the oscillation.

(d) Show that the two solutions $y_1(x) = e^x \cos x$ and $y_2(x) = e^x \sin x$ of the differential equation $y''-2y'+2y=0$ are linearly independent on the open interval I. Then find a particular solution of the above differential equation with initial condition

$$y(0)=1 \text{ and } y'(0)=5.$$

(e) Find the general solution of the Euler equation $x^2y'' + 7xy' + 25y = 0$.

SECTION – D

5. Attempt any **two** parts; each part is of **7.5** marks.

(a) Consider a disease where all those who are infected remain contagious for life. Assume that there are no births and deaths :

(i) Write down suitable word equations for the rate of change of numbers of susceptibles and infectives. Hence develop a pair of differential equations.

(ii) Draw a sketch of typical phase-plane trajectories for this model. Determine the direction of travel along the trajectories.

(b) A simple model for a battle between two army red and blue, where both the army used aimed fire, is given by the coupled differential equations –

$$\frac{dR}{dt} = -a_1B, \frac{dB}{dt} = -a_2R$$

Where R and B are the number of soldiers in the red and blue army respectively and a_1 and a_2 are the positive constants.

(i) Use the chain rule to find a relationship between R and B, given the initial numbers of soldiers for the two armies are r_0 and b_0 respectively.

(ii) Draw a rough sketch of phase-plane trajectories.

(iii) If both the army have equal attrition coefficients i.e. $a_1 = a_2$ and there are 10,000 soldiers in the red army and 8000 in blue army. Determine who wins if there is one battle between the two army.

(c) Consider the Lotka – Volterra model describing the simple predator prey model :

$$\frac{dx}{dt} = b_1X - c_1XY \quad \text{and} \quad \frac{dY}{dt} = c_2XY - a_2Y$$

where b_1, c_1, c_2, a_2 are positive constants and X and Y denotes the prey and predator populations respectively at time t.

(i) Find the equilibrium solutions of the above model.

(ii) Find the directions of trajectories in the phase plane.

This question paper contains 8 printed pages]

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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1125**

Unique Paper Code : **235201** **G**

Name of the Paper : **Differential Equation & Modeling-I**

Name of the Course : **B.Sc. (Hons.) Maths.**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Use of Scientific calculators is allowed.

Section I

1. Attempt any *three* of the following : 5+5+5

(*a*) Find particular solution of the differential equation :

$$\frac{dy}{dx} = 2xy + 3x^2 \exp\left(x^2\right), \quad y(0) = 5.$$

(*b*) Find a general solution of the differential equation :

$$x\frac{dy}{dx} - 4x^2 y + 2y \ln y = 5.$$

(*c*) Find the general solution of the differential equation :

$$\frac{d^2y}{dx^2} = \left(x + \frac{dy}{dx}\right)^2.$$

(*d*) State and prove the criterion for exactness.

2. Attempt any *two* of the following : 5+5

(*a*) Suppose that sodium pentobarbitol is used to anesthetize a dog. The dog is anesthetized when its bloodstream contains at least 45 milligrams (mg) of sodium pentobarbitol per kilogram of the dog's body weight. Suppose also that sodium pentobarbital is eliminated exponentially from the dog's bloodstream, with a half-life of 5 hours. What single dose should be administered in order to anesthetize a 50 kg dog for 1 hour ?

(*b*) Suppose that a cylindrical tank initially containing V_0 gallons of water drains (through a bottom hole) in T minutes. Use Torricelli's law to show that the volume of water in the tank after $t \leq T$ minutes is $V = V_0[1 - (t/T)]^2$.

(*c*) Consider a body that moves horizontally through a medium whose resistance is proportional to the square of the velocity, so that $dv/dt = -kv^2$. Show that :

$$v(t) = \frac{v_0}{1 + v_0 kt}$$

and that

$$x(t) = x_0 + \frac{1}{k}\ln(1 + v_0 kt).$$

Section II

3. Attempt any *two* of the following : 8+8

(*a*) Consider the problem of population in the Lake Burley Griffin. Assume the lake has a constant volume :

(*i*) Write down a differential equation describing the concentration of pollution, using V for the volume of the lake, F for Flow, $c(t)$ of concentration at time t and c_{in} for concentration of population entering the lake.

(*ii*) Further, $V = 28 \times 10^6$ m^3, $F = 4 \times 10^6$ m^3/month, find how long would it take for the lake with pollution concentration of 10^7 parts/m^3 to drop below the safety threshold (4×10^6 parts/m^3) if :

Only fresh water enters in the lake.

Water enters the lake has a pollution concentration of 3×10^4 parts/m^3.

(*b*) A public bar opens at 6 pm and is rapidly filled with clients of whom the majority are smokers. The bar is equipped with ventilators which exchange the smoke-air mixture with fresh air. Cigarette smoke contains 4% carbon monoxide and a prolonged exposure to a concentration of more than 0.012% can be fatal. The bar has a floor area of 20 m by 15 m, and a height of 4 m. It is estimated that smoke enters the room at a constant rate of 0.0006 m^3/min, and that the ventilators remove the mixture of smoke and air at 10 times the rate at which smoke is produced. The problem is to establish a wise time to leave the bar. That is, sometimes before the concentration of carbon monoxide reaches the lethal limit.

(*c*) The following model describes the levels of drugs in a patient taking a course of cold pills :

$$\frac{dx}{dt} = 1 - k_1 x, \quad x(0) = 0,$$

$$\frac{dy}{dt} = k_1 x - k_2 y, \quad y(0) = 0,$$

Where k_1 and $k_2 (k_1 > 0,\ k_2 > 0$ and $k_1 \neq k_2)$ describe rates at which the drug moves between the two sequential compartments (the GI-tract and the bloodstream) and I denotes the amount of drug released into the GI-tract in each time step. The levels of the drug in the GI-tract and bloodstream are x and y respectively :

(*i*) Show by solving the equations sequentially that the solutions are :

$$x(t) = \frac{1}{k_1}\left(1 - e^{-k_1 t}\right),\ y(t) = \frac{1}{k_2}\left[1 - \frac{1}{k_2 - k_1}\left(k_2 e^{-k_1 t} - k_1 e^{-k_2 t}\right)\right].$$

(*ii*) Find the levels of the drug in the GI-tract and the bloodstream at $t \to \infty$.

Section III

4. Attempt any *three* of the following : 6+6+6

(*a*) Solve the initial value problem :

$$3y^{(3)} + 2y^{(2)} = 0; \quad y(0) = -1, \quad y^{(1)}(0) = 0, \quad y^{(2)}(0) = 1.$$

(*b*) Use the method of undetermined coefficients to find the particular solution of the differential equation :

$$y^{(3)} + 6y^{(1)} + 13y = e^{-3x} \cos x.$$

(*c*) Use method of variation of parameter to find a particular solution of the differential equation :

$$y^{(2)} + ay = \sin 3x.$$

(*d*) A mass of 3 kg is attached to the end of a spring that is stretched 20 cm by a force of 15 N. It is set in motion with initial position $x_0 = 0$ and initial velocity $v_0 = -10$ m/s. Find the amplitude, period and frequency of the resulting motion.

Section IV

5. Attempt any *two* of the following : 8+8

(*a*) (*i*) Develop a model with three differential equations describing a predator-prey interaction, where there are two different non-competing species of prey and one species of predator.

Draw the compartmental diagram, write the word equations, define the variables appropriately and hence derive the differential equations.

(ii) The following is the model describing one prey and two predators interaction :

$$\frac{dX}{dt} = a_1X - b_1XY - c_1XZ,$$

$$\frac{dY}{dt} = a_2XY - b_2Y,$$

$$\frac{dZ}{dt} = a_3XZ - b_3Z$$

Where a_i, b_i, c_i for $i = 1, 2, 3$, are all positive constants.

Find all the possible equilibrium points.

It is possible for all three populations to coexit in equilibrium ?

(b) A model for the spread of disease, where one susceptible infected, confers life-long immunity, is given by the coupled differential equation :

$$\frac{dS}{dt} = -\beta SI, \quad \frac{dI}{dt} = \beta SI - \alpha I$$

where α and β are positive constants, S(t) denotes the number of susceptibles and I(t) denotes the number of infectives at time t :

(i) Use the chain rule to find a relation between S and I, given the initial number of susceptible and infectives are s_0 and i_0 respectively.

(ii) Find and sketch directions of trajectories in the phase plane.

(c) A simple model for a battle between two armies red and blue, where both are the armies used aimed fire, is given by the coupled differential equations :

$$\frac{dR}{dt} = -a_1 B, \quad \frac{dB}{dt} = -a_2 R$$

where R and B are the numbers of soldiers in the red and blue armies respectively, a_1 and a_2 are positive constants.

If both the armies have equal attrition coefficients $i.e.$ $a_1 = a_2$ and there are 10000 soldiers in the red army and 8000 in the blue army, determine who wins, if :

(i) There is one battle between the two armies.

(ii) There are two battles, first battle with half the red army against the entire blue army and second with the other half of the red army against the blue army survivors of the first battle.

This question paper contains 4+1 printed pages] 16/5/17

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1126**

Unique Paper Code : **235203** **G**

Name of the Paper : **Analysis-II [MAHT-202]**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *three* parts of each question.

All questions are compulsory.

1. (*a*) Use the $\in - \delta$ definition of the limit to show that :

$$\lim_{x \to c} x^2 = c^2, \ c \in \mathbb{R}.$$

(*b*) Let $A \subseteq \mathbb{R}$, let $f : A \to \mathbb{R}$ and let c be a cluster point of A. If

$$\lim_{x \to c} f(x) < 0$$

then prove that there exists a neighbourhood $V_\delta(c)$ of c such that $f(x) < 0$ for all $x \in A \cap V_\delta(c)$, $x \neq c$.

P.T.O.

(*c*) Using Sequantial Criterion for limits, prove that :

$$\lim_{x \to 0} \sin(1/x^2)$$

does not exist in R.

(*d*) Using definition, prove that :

(*i*) $\lim_{x \to 0} \left(\frac{1}{x^2}\right) = \infty$

(*ii*) $\lim_{x \to \infty} 1/x = 0.$ 5,5,5,5

2. (*a*) Prove that a function $f : A \to R$ is continuous at a point $c \in A$ if and only if for every sequence (x_n) in A that converges to c, the sequence $(f(x_n))$ converges to $f(c)$.

(*b*) Let $A \subseteq R$, let $f : A \to R$ be continuous at a point $c \in R$. Show that for any $\in > 0$ there exists a neighbou..ood $_\delta(c)$ of c such that if $x, y \in A \cap V_\delta(c)$ then :

$$|f(x) - f(y)| < \in.$$

(c) Let f and g be continuous from R to R and suppose that $f(r) \geq g(r)$ for every rational numbers then prove that $f(x) \geq g(x)$ for all $x \in$ R.

(d) State Bolzano's Intermediate value theorem and hence prove that $xe^x = 2$ for some x in $[0, 1]$. 5,5,5,5

3. (a) Let I = $[a, b]$ be an interval and let f : I → R be continuous on I. Then prove that f is uniformly continuous on I.

(b) Show that the function $f(x) = 1/x$ is uniformly continuous on $[a, \infty[$, $a > 0$ but is not uniformly continuous on $]0, \infty[$.

(c) Given that the function $f(x) = x^3 + 2x + 1$ for $x \in$ R has an inverse f^{-1} on R, find the value of $(f^{-1})'(y)$ at the points corresponding to $x = 0, 1$.

(d) Prove that if f : I → R has a derivative at $c \in$ I, then f is continuous at c. Is the converse true ? Justify your answer. 5,5,5,5

4. (*a*) Let c be an interior point of the interval I at which $f : I \to R$ has a relative extremum. If the derivative of f at c exists then prove that :

$$f'(c) = 0.$$

Can f has a relative extremum at 'c' without being differentiable at 'c' ? Justify.

(*b*) Using the mean value theorem, prove that :

$$\frac{x-1}{x} < \log(x) < x - 1 \text{ for } x > 1.$$

(*c*) Find the point of relative extrema of the function :

$$f(x) = x(x - 8)^{1/3} \text{ for } 0 < x < 9.$$

(*d*) State Darboux theorem. Let $f : [0, 2] \to R$ is continuous on $[0, 2]$ and differentiable on $]0, 2[$ and that $f(0) = 0$, $f(1) = 2$, $f(2) = 2$ then show that there exists $c \in]0, 2[$, such that :

$$f'(c) = 3/2.$$

5,5,5,5

5. (a) Obtain Maclaurin's series expansion for the function $\sin(4x)$.

(b) Using Taylor's theorem prove that :

$$1 + \frac{x}{2} - \frac{x^2}{8} < \sqrt{1 + x} < 1 + \frac{x}{2} \text{ for } x > 0.$$

(c) Define Radius of convergence of the power series :

$$\sum_{n=0}^{\infty} a_n x^n.$$

Find the Radius of convergence and the exact interval of convergence for the power series :

$$\sum_{n=1}^{\infty} \frac{x^n}{n^2}.$$

(d) Check which of the following functions are convex :

(i) $|x|, x \in [-2, 5]$

(ii) $ax^3 + 2x + 3, a < 0, x \in [-1, 1].$ 5,5,5,5

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This question paper contains 7 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1127**

Unique Paper Code : **235204** **G**

Name of the Paper : **Probability and Statistics—MAHT 203**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

In all there are *six* questions.

Question No. **1** is compulsory and

it contains five parts of **3** marks each.

In Question No. **2** to **6**, attempt any *two* parts from three parts.

Each part carries **6** marks.

Use of Scientific calculator is allowed.

1. (*a*) If C_1 and C_2 are events such that $C_1 \subseteq C_2$, then prove that $P(C_1) \leq P(C_2)$.

(*b*) Let X be a random variable with mean μ, variance σ^2 and μ_2' as the second moment about the origin, then show that $\sigma^2 = \mu_2' - \mu^2$.

(*c*) Let X have the probability density as follows :

$$f(x) = \begin{cases} k\, e^{-3x} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Find k and $P(0.5 \leq X \leq 1)$.

(*d*) If the probability density of X is given by :

$$f(x) = \begin{cases} 2(1 - x) & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Show that :

$$E(X^r) = \frac{2}{(r + 1)(r + 2)}.$$

(*e*) A five card poker hand dealt from a deck of 52 playing cards is said to be a full house if it consists of three of a kind and a pair. If all the five card hands are equally likely, what is the probability of being dealt a full house ?

2. (*a*) Let $\{C_n\}$ be an increasing sequence of events, then show that :

$$\lim_{n \to \infty} P(C_n) = P(\lim_{n \to \infty} C_n) = P\left(\bigcup_{n=1}^{\infty} C_n\right).$$

(*b*) Define a negative binomial distribution with parameters k and θ. If the probability is 0.40 that a child exposed to a certain contagious disease will catch it, what is the probability that the tenth child exposed to the disease will be the third to catch it ?

(*c*) If A, B, C are any three events in a sample space, then show that :

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

3. (*a*) Define a normal distribution and show that its moment generating function is given by :

$$M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

(*b*) For a binomial distribution find its moment generating function and hence find its mean and variance.

(*c*) Find the moment generating function of the geometric distribution with parameter θ and use it to show that its mean is $\frac{1}{\theta}$ and variance is $\frac{1-\theta}{\theta^2}$.

4. (*a*) Let X_1 and X_2 be two random variables with joint pdf as :

$$f(x_1, x_2) = \begin{cases} 4x_1x_2 & 0 < x_1 < 1, \\ & 0 < x_2 < 1, \\ 0 & \text{elsewhere} \end{cases}$$

Is $E(X_1X_2) = E(X_1)\,E(X_2)$?

(*b*) Let the random variables X and Y have a joint pdf :

$$f(x, y) = \begin{cases} x + y & 0 < x < 1,\ 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the correlation coefficient of X and Y.

(c) Let :

$$f(x_1, x_2) = \begin{cases} 21x_1^2x_2^3 & 0 < x_1 < x_2 < 1 \\ 0 & \text{otherwise} \end{cases}$$

be the joint pdf of X_1 and X_2.

Find :

(i) the conditional mean of X_1 given $X_2 = x_2$, $0 < x_2 < 1$, and

(ii) the distribution of $Y = E(X_1|X_2)$.

5. (a) Find the marginal density of X and Y if the pair of random variables (X, Y) has a bivariate normal distribution.

(b) If the regression of Y on X is linear, then show that :

$$\mu_{Y|x} = \mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x - \mu_1).$$

(c) Given the two random variables X and Y that have the joint density :

$$f(x, y) = \begin{cases} x \cdot e^{-x(1+y)} & \text{for } x > 0 \text{ and } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the regression equation of Y on X and sketch the regression curve.

6. (a) If an individual is in state 1 (lower income class) then there is a probability of 0.65 that any offspring will be in the lower income class, a probability of 0.28 that offspring will be in state 2 (middle income class), and a probability of 0.07 that offspring will be in the state 3 (upper income class).

Write the transition matrix P for the said process. If a parent is in state 3 (upper income class), find the probability that a grandchild will be in state 2 (middle class).

(*b*) State and prove Chebyshev's Theorem.

(*c*) Let X be the number of times that a fair coin, flipped 40 times, lands heads. Find the probability that X = 20. Use the normal approximation and then compare it to the exact solution.

(18) 09/5/17

Your Roll No...............

S. No. of Ques. Paper : 1189

Unique Paper Code : 217281

Name of Paper : Chemistry Paper III (CHCT-101)

Name of Course : B.Sc. (Hons.) Mathematics Concurrent Course-III/ B.Sc. Mathematical Science

Semester : II

Duration : 3 hrs

Maximum Marks : 75

G

(Write your Roll No. on the top immediately on receipt of the question paper)

*Attempt **three** questions from Section A and **three** questions from Section B. Use separate answer sheets for Sections A and B. Questions should be numbered in accordance to the number in the question paper. Calculators may be used.*

SECTION A

Attempt three questions from this section.

1.(a) On the basis of MO theory, N_2 molecule is diamagnetic, while O_2 molecule is paramagnetic. Explain.

(b) What are Fajan's rules? Explain, giving suitable example.

(c) $BaSO_4$ is insoluble in water, whereas Na_2SO_4 is soluble in water. Explain.

(d) Represent the splitting of *d*-orbitals in a square planar field.

4,3,3,2½

2. Calculate the lattice energy of CsCl using the following data:

$Cs(s) \xrightarrow{S} Cs(g)$ $\Delta H = 79.9$ kJ/mol

$Cs(g) \xrightarrow{I} Cs^+(g)$ $\Delta H = 374.05$ kJ/mol

$$Cl_2(g) \xrightarrow{D} 2Cl\cdot(g) \qquad \Delta H = 241.84 \text{ kJ/mol}$$

$$Cl\cdot(g) \xrightarrow{EA} Cl^-(g) \qquad \Delta H = -397.90 \text{ kJ/mol}$$

$$Cs(g) + \tfrac{1}{2} Cl_2(g) \xrightarrow{\Delta H_f} CsCl \qquad \Delta H = -623.00 \text{ kJ/mol}$$

(b) Account for the following: BeF_2 is linear, while SF_2 is angular in shape.

(c) NaCl is ionic, but NaI is predominantly covalent. Explain.

(d) Write the main postulates of VSEPR theory.

4,3,3,2½

3.(a) Predict the final products formed in the following reactions on the basis of *trans* effect (with explanation):

(i) $[PtCl_4(NH_3)]^- + NO_2^- \rightarrow$?

(ii) $[PtCl_4(NO_2)]^{2-} + NH_3 \rightarrow$?

(b) Draw the resonance structures of CO_3^{2-} ion.

(c) What is the Jahn-Teller effect?

(d) On the basis of hybridization, predict the shapes of the following molecules:

XeF_6, $ClOF_3$, IO_4^-

4,2½,3,3

4. (a) Comment on Schottky and Frenkel effects (with suitable examples).

(b) What is the concept of multiplicity rule? Explain.

(c) The electron transfer from $[Co(NH_3)_6]^{2+}$ to $[Co(NH_3)_6]^{3+}$ is slower than from $[Fe(CN)_6]^{4-}$ to $[Fe(CN)_6]^{3-}$. Explain.

(d) How will you account for the smaller bond order of NO compared to NO^+ on the basis of MO theory?

4,2½,3,3

SECTION B

Attempt three questions from this section.

5. Explain why:

(a) Aliphatic amines are stronger bases than the aromatic amines.

(b) Vinyl carbocation is less stable than the corresponding alkyl carbocation.

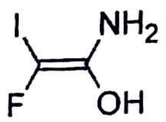
(c) *p*-Nitrophenol is more acidic than *o*-nitrophenol.

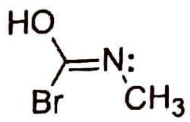
(d) Chair conformation of cyclohexane is more stable than boat conformation.

(e) Diethyl ether has lower boiling point and lower water solubility as compared to that of 1-butanol.

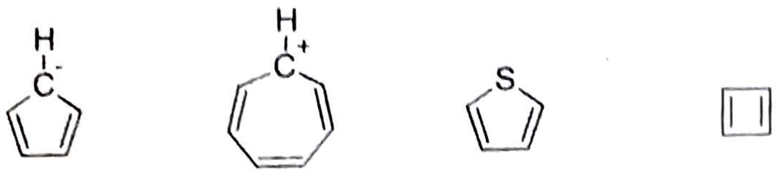
5 × 2½ = 12½

6. (a) Assign *E/Z* configuration to the following compounds:





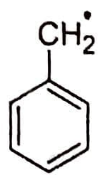
(b) Which of the following compounds are aromatic and why?



(c) Draw all the possible stereoisomers of tartaric acid [COOH-CHOH-CHOH-COOH]. Explain their relationships with each other. Which of these are optically active and which are optically inactive?

4,4,4½

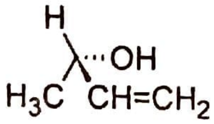
7. (a) Draw resonance structures of benzyl radical

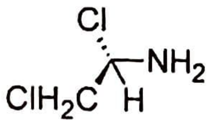


(b) *o*-Bromoanisole and *m*-bromoanisole, on treatment with iodamide in liquid ammonia, give the same product. Name the product and explain its formation.

(c) Differentiate between natural and synthetic rubber. Explain, giving their synthesis.

(d) Assign *R/S* configuration to the following compounds:





2 ½,4,3,3

8. Complete the following reactions and also indicate the name of the reaction:

(a) $C_6H_5CHO + (CH_3CO)_2O \xrightarrow[\Delta]{CH_3COONa}$?

(b) $C_6H_5COCH_3 + HCHO + C_2H_5NHC_2H_5 \rightarrow$?

(c) $CH_3CH{=}CH_2 + HBr \xrightarrow{\text{Peroxide}}$?

(d) $2\ HCHO \xrightarrow[\Delta]{NaOH(aq)}$?

(e) $CH_3COOC_2H_5 \xrightarrow{C_2H_5ONa}$?

5 × 2½

(19)

This question paper contains 3 printed pages.

Your Roll No. ....11/5/17........

Sl. No. of Ques. Paper	: 6482	G
Unique Paper Code	: 232471	
Name of Paper	: Citizenship in a Globalizing World	
Name of Course	: B.Sc. (Hons.) Mathematics	
Semester	: IV	
Duration	: 3 hours	
Maximum Marks	: 75	

(Write your Roll No. on the top immediately on receipt of this question paper.)

NOTE:— *Answers may be written either in English or in Hindi; but the same medium should be used throughout the paper.*

टिप्पणी:— इस प्रश्नपत्र का उत्तर अंग्रेज़ी या हिन्दी किसी एक भाषा में दीजिए; लेकिन सभी उत्तरों का माध्यम एक ही होना चाहिए।

Attempt any four questions.
All questions carry equal marks.

किन्हीं चार प्रश्नों के उत्तर दीजिए।
सभी प्रश्नों के अंक समान हैं।

1. The concept of citizenship has evolved with time. Explain.

P. T. O.

नागरिकता का सिद्धांत समय के साथ विकसित हुआ है। व्याख्या कीजिए।

2. Discuss the classical concept of citizenship.

नागरिकता की शास्त्रीय अवधारणा की विवेचना कीजिए।

3. Examine T.H. Marshall's views on citizenship.

टी.एच. मार्शल के नागरिकता के सिद्धांत का परीक्षण कीजिए।

4. What is multiculturalism? Discuss it with reference to Will Kymlica's views on multicultural society.

बहुसंस्कृतिवाद से आप क्या समझते हैं? विल किमलिका के बहुसंस्कृतिवादी समाज का अवलोकन कीजिए।

5. What is globalization? Critically examine its effects on State Sovereignty.

वैश्वीकरण क्या है? राज्य की सम्प्रभुत्ता पर इसके प्रभाव का आलोचनात्मक परीक्षण कीजिए।

6. Some developed nations are making its immigration laws more stringent. Will this affect the pace of globalization in the world?

कुछ विकसित राज्य आवर्जन कानूनों को और अधिक कठोर बना रहे हैं। वैश्वीकरण पर इसके प्रभाव की चर्चा कीजिए।

7. The concept of citizenship is shrinking in a globalized world. Discuss.

वैश्वीकरण के युग में नागरिकता का सिद्धांत धूमिल होता जा रहा है। विवेचना कीजिए।

8. Write short notes on any *two* of the following:

(a) Social Exclusion

(b) Global Justice

(c) Human Rights

(d) Aristotle's concept of citizenship.

निम्नलिखित में से किन्हीं **दो** पर संक्षिप्त टिप्पणी लिखिए:

(a) सामाजिक बहिष्करण

(b) वैश्विक न्याय

(c) मानव अधिकार

(d) अरस्तु का नागरिकता का सिद्धांत।